

Investigation on Measurement Stability of Fabry-Pérot Angle Sensor using Mode-locked Femtosecond Laser

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Precision angular position measurements for ultra-positioning systems are increasingly important for a higher positioning accuracy or the elimination of Abbe error. In this research, the absolute angle measurements method using Fabry-Pérot angle sensor configured with Fabry-Pérot etalon, and a mode-locked femtosecond laser is proposed. Fabry-Pérot etalon is a multi-beam interference-based optical resonator with many applications for various measurements of quantities. The angle can be absolutely measured by evaluating the angle of incidence from the peak wavelengths of the broadband spectral response. However, an oblique incidence to the Fabry-Pérot etalon and its limited size cause loss of interference beams, which is an issue for improving angle measurement performance due to a decrease in peak detection accuracy. To address this issue, the Fabry-Pérot angle sensor employs a spectral division method. The spectral division method divides two types of spectral responses obtainable from the Fabry-Pérot etalon. This method improves angle measurement accuracy by greatly amplifying the intensities of the spectrum near the peaks based on the division of spectra 180 degrees out of phase. In this report, absolute angle measurement results of the Fabry-Pérot angle sensor configured with FP etalon and its stability under room conditions are investigated. The influence of the fluctuations on mode-locked femtosecond laser source power on the measurement accuracy is observed by repeated measurement and evaluation of the angle.

NOMENCLATURE

θ = reflection angle at the Fabry-Perot etalon cavity
 n = refractive index of Fabry-Perot etalon cavity
 d = length of cavity
 F = coefficient of Finesse
 λ = Wavelength
 I_0 = Intensity of mode-locked femtosecond laser source
 m = Integer multiples

1. Introduction

The development of ultra-precision positioning technologies of motion stages is increasingly important in the recent precision manufacturing system. For the precision positioning systems that do not satisfy the Abbe principle, the detection of angular motion errors of the motion stage is required to improve the positioning accuracy [1]. For example, an angular motion error of several arcseconds could cause a sub-micrometric positioning error in the general case of the

precision manufacturing system [2]. For this purpose, precision angle measurement is one of the important tasks in achieving nanometric positioning accuracy. Laser autocollimators are commercially available and well-used angle sensors that offer accuracy typically sub-arcseconds [3]. Despite its performance, geometric optics-based measurement of autocollimators normally requires a large sensor size for sufficient measurement resolution. The relative measurement of the angle can be also a drawback in the improvements of manufacturing efficiency.

For a new angle sensor for an absolute angle measurement satisfying those requirements, this research investigates on a Fabry-Perot (FP) angle sensor using a FP etalon and a mode-locked femtosecond laser [4]. FP etalons are multi-beam interference based optical resonators consisting of two parallel half mirrors. FP etalons have many applications to absolute measurements of quantities by employing broadband light sources [5]. The angle measurements using FP etalons can be operated by determining the incident angle of the light source to the FP etalon surface. This angle can be obtained from the peaks of the interference spectrum which can be expressed as the relationship with integer multiples of the peak wavelength. Meanwhile, spatial interference at FP etalons and considering such a long optical

path of the precision positioning system demands broadband light source to have good directionality and enough coherence length. Employing the mode-locked femtosecond laser as the light source is advantageous for this purpose as it has good spatial coherence length and a wide spectral range for multi-wavelength interferometry [6].

To produce the spectral response of the FP etalon, parallel beams from the FP etalon cavity are focused and collected to be detected by an optical spectrum analyzer. However, the loss of interference beams at the FP etalon cavity is inevitable in the angle measurement due to the oblique incidence of the ray to the FP etalon surface. This causes spectral widths to be widened, which decreases peak detection accuracy. Therefore, this research proposed a spectral division method, which divides two obtainable interference spectra with 180 degrees out of phase [4]. Since the angle measurement accuracy is associated with the peak detection accuracy, this method can improve the sensor accuracy by greatly amplifying the peak intensities. In this report, the measurement stability of the Fabry-Perot angle sensor when employing air-gapped cavity-typed etalon is investigated. Repeated measurements for both obtainable interference spectra were conducted for 100 times in the room temperature condition. Absolute angles were evaluated from obtained all spectra and those were compared to check the measurement stabilities. The power fluctuations were also simultaneously measured during the repeated angle measurements to check the influences on the measurement stability.

2. Principle

2.1 Absolute angle measurements

Fig. 1 shows a schematic of the angular motion of the FP etalon and its optical configuration as the FP angle sensor. Repeated reflections in the FP etalon cavity are produced by injecting a collimated mode-locked femtosecond laser source into the FP etalon with a non-zero incident angle. Two types of interference beams are then produced from both surfaces of the FP etalon, which are denoted as transmittance beams (Tr) and reflectance beams (Re). Multi-beam interferences of both groups of beams can be occurred by focusing those beams by focusing lenses. The resultant responses of the interference can be observed as fringe spectra. The fringe spectra of Tr and Re are determined by the FP etalon geometry and the wavelength of a laser, which can be expressed by the following equations [4]:

$$Tr = \frac{I_0}{1 + F \sin^2(2\pi nd \cos \theta / \lambda)} \quad (1)$$

$$Re = I_0 - Tr = \frac{I_0 F \sin^2(2\pi nd \cos \theta / \lambda)}{1 + F \sin^2(2\pi nd \cos \theta / \lambda)} \quad (2)$$

The fringe spectra of Tr and Re can be detected by focusing and coupling those groups of beams to single-mode optical fibers connected to an optical spectrum analyzer (OSA). Values of λ satisfying $Tr = 1$ and $Re = 0$, or denoted as λ_{peak} , are observed as peaks of the fringe spectrum. These values can be evaluated by employing peak detection methods of the spectrum, as shown in Fig. 2. Thus, the angular position can be evaluated by defining θ value satisfying $Tr = 1$ and $Re = 0$ at peak wavelengths, which can be expressed by the

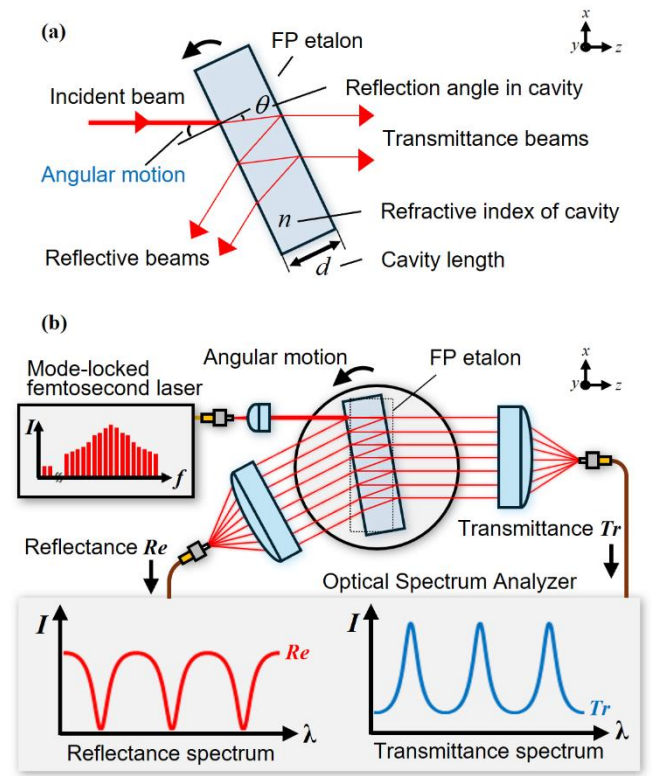


Fig. 1 A schematic of (a) an angular motion of a FP etalon and (b) a FP angle sensor using mode-locked femtosecond laser

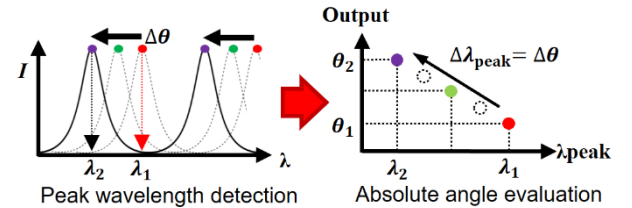


Fig. 2 Absolute angle evaluations using peak of the fringe spectrum

following equation:

$$\theta_{\text{Output}} = \arcsin(n \sin \theta) = \arcsin \left[n \sin \left(\arccos \frac{m \lambda_{\text{peak}}}{2nd} \right) \right] \quad (3)$$

Angular motion θ_{output} can be detected by using the reflection angle θ and the refractive index of the FP etalon cavity based on Snell's law. However, Eq. 3 can be ignored when air-gapped typed FP etalon is used. Multiple peak wavelengths containing equal angle information can be obtained at a single measurement of the wide spectral range of the mode-locked femtosecond laser. By taking this advantage, the final output angular position can be the average of all angle data from obtained peak wavelengths for reducing measurement error and uncertainty.

2.2 Spectral division method

The proposed spectral division method is operated by dividing detected intensities of the Tr spectrum by the corresponding Re

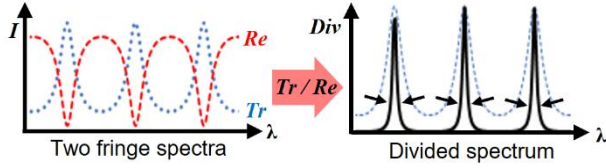


Fig. 3 A schematic of the proposed spectral division method

spectrum, as shown in Fig. 3. In this paper, the obtained spectrum by the spectral division method is named a divided spectrum (**Div**). The equation for **Div** is obtained by dividing Eq. 1 by Eq. 2, which can be expressed as the following equation [4]:

$$\text{Div} = \frac{Tr}{Re} = \frac{1}{F \sin^2(2\pi nd \cos \theta / \lambda)} \quad (4)$$

Angular position can be evaluated by peak detection of **Div**, equally with the method of **Tr** and **Re**. It can be seen from the equation that **Div** converges to infinity as the sine value satisfies zero. This represents a great amplification of the peak. An elimination of I_0 in the equation indicates reductions of influences from the laser source spectrum, which is also beneficial in improving peak detection accuracy.

3. Experiments

3.1 Experiment setup

Fig. 4 shows a constructed optical setup of the FP angle sensor for the measurement experiment. A commercially available mode-locked femtosecond laser source (Menlo C-Fiber HP) with a central wavelength of 1550 nm and an OSA (Yokogawa, Co AQ6370C) were employed. The interference beams from the FP etalon were collected by precisely focusing to single-mode optical fibers (Thorlabs, P1-1550A-FC) connected to the OSA. A FP etalon ($n = 1$, $d = 750 \mu\text{m}$, $F = 31.1$) was mounted on a precision rotary stage. The initial angle was aligned to be approximately 35400 arcseconds (9.833 degrees) based on the rotary stage reading. The power fluctuation of the mode-locked femtosecond laser source was measured by using an optical powermeter (Thorlabs, S140C). The temperature of the experiment room was controlled at 26 °C.

3.2 Three fringe spectra observations

The fringe spectra were obtained at the aligned angle position with the bandwidth of 1545 nm ~ 1555 nm. Fig. 5 (a) shows the obtained fringe spectra of **Tr** and **Re**. The total intensities were normalized by the local maxima of the spectra. The entire inclinations on the intensities of peaks were due to influences of the laser source spectrum. Both obtained **Tr** and **Re** were verified to be completely out of phase because the local maxima of **Tr** and local minima of **Re** corresponded.

The spectral division method using obtained **Tr** and **Re** was carried out to evaluate the fringe spectrum of **Div**. Fig. 5 (b) shows the obtained fringe spectrum of **Div** with **Tr** for the spectral width comparison. All 6 peak intensities of **Div** were converged to nearly 1, which indicates the inclination of the peak intensities observed at previous spectra were eliminated by the spectral division. This also verified the elimination

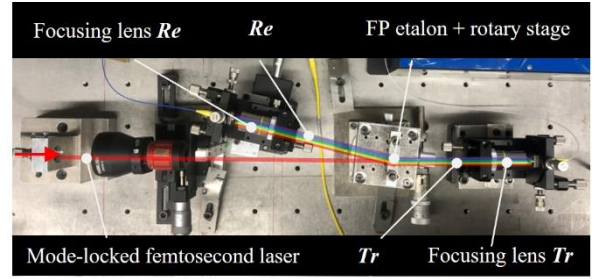


Fig. 4 A schematic of the proposed spectral division method

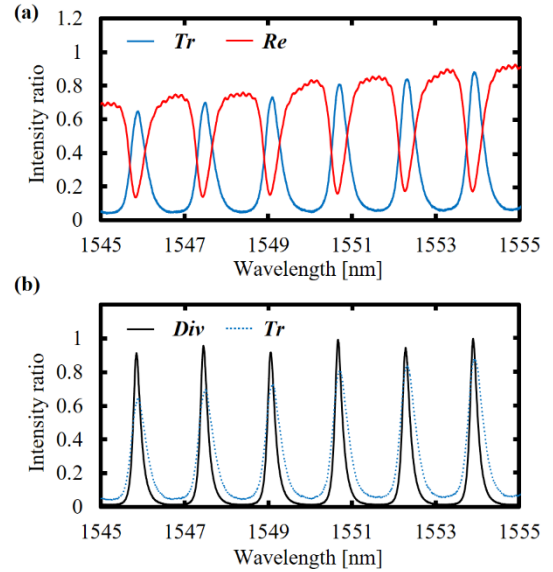


Fig. 5 Obtained fringe spectra of (a) **Tr** and **Re** at aligned angle position of 30 degrees and (b) evaluated **Div** using the spectral division method.

of I_0 in Eq. 4. The spectral widths for three spectra at an intensity ratio of 0.5 (full-width half maximum: FWHM) were evaluated for the verification of the spectral division method. The FWHMs for **Tr** and **Re** were evaluated to be 0.480 nm and 0.489 nm, respectively. The FWHM for **Div** was then reduced to 0.248 nm. This verified the validity of the spectral division method by narrowing the spectral widths by approximately 50% in the normalized spectrum.

3.3 Repeated measurements result

Repeated measurements for both fringe spectra of **Tr** and **Re** were conducted for 100 times in the experiment room condition. The temperature was controlled to be at 26°C by air conditioner. Peak wavelengths of each spectrum were evaluated by a centroid method. Denote sampling wavelengths as I_i and those intensities as λ_i , the centroid method for peak evaluations is expressed as following equations [7]:

$$\lambda_{peak} = \frac{\sum \lambda_i \cdot I_i}{\sum I_i} \quad (\text{for } Tr, Div \geq 0.5 \cdot \text{Local maximum}) \quad (5)$$

$$\lambda_{peak} = \frac{\sum \lambda_i \cdot I_i}{\sum I_i} \quad (\text{for } Re \leq 0.5 \cdot \text{Local maximum}) \quad (6)$$

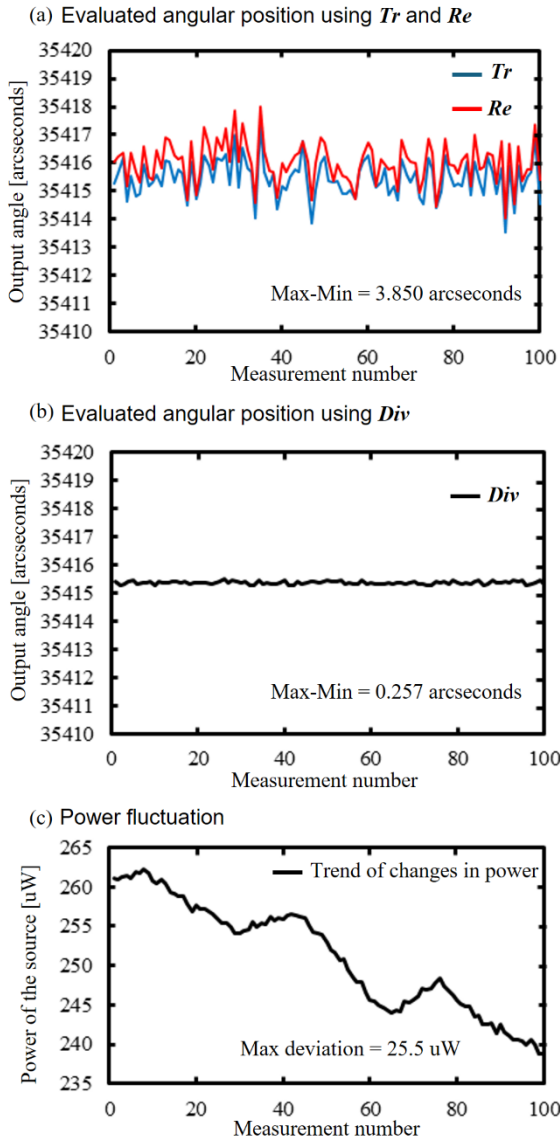


Fig. 6 (a) Evaluated angular position from *Tr* and *Re* of the repeated measurements, (b) Evaluated angular position from *Div* and (c) power fluctuations during the measurements

Based on the peak evaluation results, angular positions were then evaluated by using Eq. 3. Power fluctuations of the mode-locked femtosecond laser source were also simultaneously measured during the repeated angle measurements.

Fig. 6 (a) shows the evaluated angle position for fringe spectra of *Tr* and *Re*. The maximum angle evaluation error from both spectra was 3.850 arcseconds. Corresponding angle evaluation results of *Div* is shown in Fig. 6 (b). It can be seen from the figure that angle evaluation error was improved to 0.257 arcseconds, which verified the improvements using spectral sharpening of the spectral division method. Fig. 6 (c) shows the measured power fluctuations of the laser source during the repeated measurements. The power of the laser source fluctuated for 25.5 μ W and it was continuously decreasing until the 100th measurement. Despite this fluctuation in the power, angle evaluation results of all three spectra (*Tr*, *Re* and *Div*) fluctuated near the aligned angular position regardless of the power degradation trend.

4. Conclusion

The measurement stability of the FP angle sensor using an air-gapped-typed FP etalon and the mode-locked femtosecond laser has been investigated by repeated angle measurements. The proposed spectra division method improved the angle evaluation error from a maximum of 3.850 arcseconds to 0.257 arcseconds. This improvements on measurement accuracy were achieved by spectral width sharpening and elimination of the laser source spectrum effects using the spectral division method. The power fluctuation of the mode-locked femtosecond laser was not critically influential to the angle evaluation for all three spectra (*Tr*, *Re* and *Div*). This is because the normalization based on the local maximum intensity is operated during the peak detection method, which makes peak positions of the *Tr* and *Re* comparatively to be stable. The measurement stability further improved when using the fringe spectrum of *Div* due to the elimination of the laser source spectrum during the spectral division method. In future work, experiments with various optical configuration of the FP angle sensor will be investigated for the design optimization.

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